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Exercise of filtering
Digitization is performed by sampling and quantization.
Sampling and quantization

One-dimensional explanation

**Sampling**

At each sampling point, continuous value $f(x)$ is approximated by a proper integer.

**Quantization**

Quantization is usually done into $2^n$ levels (n corresponds to the # of bits)

- 8bits ⇒ 256 levels
- 10bits ⇒ 1024 levels
- ...

At each sampling point, continuous value $f(x)$ is approximated by a proper integer.

**Signal intensity**

$f(x)$

At each sampling point, continuous value $f(x)$ is approximated by a proper integer.

**Quantization level**: 10bits ⇒ 1024 levels

At each sampling point, continuous value $f(x)$ is approximated by a proper integer.

**Signal intensity**

$f(x_i)$

0 255

At each sampling point, continuous value $f(x)$ is approximated by a proper integer.

**Quantization level**: 10bits ⇒ 1024 levels

At each sampling point, continuous value $f(x)$ is approximated by a proper integer.
Sampling theorem

If the rapidly oscillating wave is sampled in coarse interval, how is sampled data?

Original continuous signal
Sampled signal

Apparent signal becomes a slow wave.

Sampling theorem

If the sampling satisfies the following condition, original continuous signal can fully be recovered from the sampled data.

$$\Delta x \leq \frac{1}{2u_{max}}$$

$\Delta x$: sampling pitch
$u_{max}$: Maximum frequency that the original continuous signal includes.

Just the case that $\Delta x = \frac{1}{2u_{max}}$
Two-dimensional sampling and quantization

Sampling → quantization

\[ f_{ij} \]

Quantization:

- 0
- 255

Sampling:

\[ x \rightarrow y \]
Simulation of sampling and quantization
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Tone mapping (point processing)

General expression of tone mapping

\[ g(x, y) = T[f(x, y)] \]

- \( f(x, y) \): input image
- \( g(x, y) \): processed image
- \( T[] \): tone mapping operator

Tone mapping is performed pixel by pixel.

Examples of tone mapping
- Linear transformation
- Nonlinear transformation using gamma
- Histogram equalization

Example of tone mapping
Increase/decrease of brightness

general expression \( g(x, y) = T[f(x, y)] \)

A linear transformation
\[ g(x, y) = af(x, y) + b \]

Example (from demo of MATLAB)
Increase/decrease of contrast

general expression \[ g(x, y) = T[f(x, y)] \]

A linear transformation
\[ g(x, y) = af(x, y) + b \]

Example (from demo of MATLAB)
Increase/decrease of contrast

General expression: \( g(x, y) = T[f(x, y)] \)

A non-linear transformation

\[ g(x, y) = \left[ f(x, y) \right]^\gamma \]

Example (from demo of MATLAB)
Histogram equalization

Example (from demo of MATLAB)
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Noise reduction:
When multiple images with a fixed foreground and random noise are available, averaging those images produces a noise-reduced image.

Obtained image

\[ g_1(x, y) = f(x, y) + n_1(x, y) \]
\[ g_2(x, y) = f(x, y) + n_2(x, y) \]
\[ \vdots \]
\[ g_m(x, y) = f(x, y) + n_m(x, y) \]

Processing

\[ \bar{g}(x, y) = \frac{1}{m} \sum_{i=1}^{m} g_i(x, y) \]

Effect

\[ \bar{g}(x, y) = \frac{1}{m} \sum_{i=1}^{m} f(x, y) + \frac{1}{m} \sum_{i=1}^{m} n_i(x, y) \]
\[ = f(x, y) + \frac{1}{m} \sum_{i=1}^{m} n_i(x, y) \]

Noise components are averaged.

Variance of noise decreases as \( \frac{\sigma^2}{m} \)
\( \Leftrightarrow \) SD of noise decreases as \( \frac{\sigma}{\sqrt{m}} \)
Image addition - example -

No addition

Average of 10 images
Image subtraction

In a model that subjects of interest (foreground) is added to the background, if an image of background only is available, the subjects are enhanced by subtracting the background image from foreground + background Image.

**Obtained image**

\[
g(x, y) = f(x, y) + b(x, y)
\]

and

\[
b(x, y)
\]

**Processing**

\[
h(x, y) = g(x, y) - b(x, y)
\]

**Effect**

\[
h(x, y) = [f(x, y) + b(x, y)] - b(x, y)
\]

\[
= f(x, y)
\]

**Example**

\[
g(x, y)
\]

\[
\]

\[
\]

\[
h(x, y)
\]
In a model that subjects of interest (foreground) is illuminated non-uniformly, if the illumination distribution is obtained as an image, non-uniformity is corrected by dividing the image of the subject by the illumination distribution.

\[
g(x, y) = i(x, y) f(x, y)
\]
and

\[
i(x, y)
\]

\[
h(x, y) = \frac{g(x, y)}{i(x, y)} = f(x, y)
\]

Example

\[
g(x, y) / i(x, y) = f(x, y)
\]
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